Critical Configurations For Radial Distortion Self-Calibration

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VisualSFM

- A Visual Structure from Motion System, 2011
  - SiftGPU + Multicore BA + Fast SfM + GUI
  - Used in aerial survey, geology, archaeology, VFX, 3D printing, etc.
  - Reconstruction failures are often well understood.
Question from Thomas Gröninger

• A typical aerial image capture
  – UAV flies at roughly constant height
  – Camera pointing downward (nadir)
  – Un-calibrated GoPro camera

• Distorted reconstruction, why?
  – Ground should be roughly **FLAT**
  – Incorrect radial distortion estimation
Ambiguities in 3D Reconstruction

• Extensive studies for perspective cameras
  – For calibrated reconstruction from image velocity or two views, critical surfaces are *ruled quadrics* [Horn 1987, Maybank 1993].
  – Critical motions exist for self-calibration, for example, planar motion and orbital motion [Sturm 1997, Sturm 1999, Kahl et al. 2000, etc.].

• Little study for radial distortion self-calibration
  – Parallel feature displacements and camera motion under pure translation. [Mičušík et al. 2006]
Critical Surfaces

• Horn, *Motion fields are hardly ever ambiguous*, 1987

  – Given a translational speed $t$ and rotational speed $\omega$, the image velocity is a function of $p$ and $Z$.

    $$ p' = V(t, \omega, p, Z) $$

  – For two motion $\{t_1, \omega_1\}$ and $\{t_2, \omega_2\}$, the surface pair $\{Z_1, Z_2\}$ that produce the same image velocity satisfy:

    $$ V(t_1, \omega_1, p, Z_1) = V(t_2, \omega_2, p, Z_2) $$

  – These *critical* surfaces are ruled quadrics.
The Problem

Given two cameras with

- **Different radial distortions** and
- Possibly different motions,

What surfaces can produce the same motion field?
Radial Distortion

- Central and centered radial distortion
  
  - Using an implicit radial distortion function $f(r^2)$
  
  - Not limited to specific radial distortion parameterization
  
  - Works for central omni-directional cameras
Critical Surfaces

• Image velocity in the undistorted image
  \[ (Fp)' = (F + 2F'pp^T)p' \]

• Consider the following two configurations:
  - 1\textsuperscript{st} camera with motion \{t_1, \omega_1\} without radial distortion
  - 2\textsuperscript{nd} camera with motion \{t_2, \omega_2\} and distortion function \( f \)

Solve for the critical surface pair \( Z_1 \) and \( Z_2 \):

\[ V(t_2, \omega_2, Fp, Z_2) = (F + 2F'pp^T) \quad V(t_1, \omega_1, p, Z_1) \]

Undistorted 2\textsuperscript{nd} image

1\textsuperscript{st} image
Critical Surface Pair

• The two corresponding surfaces

\[ Z_1 = \frac{-2f'(t_1 \cdot \hat{z})(p^T p)(t_2 \times \hat{z})^T p + 2f'p^T t_1(t_2 \times \hat{z})^T p - (t_2 \times Ft_1)^T Fp}{((Fp) \times \omega_2 - F(p \times \omega_1)) \cdot (t_2 \times Fp) + 2f'(p^T p)p^T (\omega_1 \times \hat{z})(t_2 \times \hat{z})^T p} \]

\[ Z_2 = \frac{Z_1 \ t_2 \cdot (\hat{z} \times p)}{(Ft_1 - ((Fp) \times \omega_2 - F(p \times \omega_1))Z_1) \cdot (\hat{z} \times p)} \]

– The critical surfaces in Horn’s paper can be obtained by using \( f = 1 \) and \( f' = 0 \);

– Complicated surfaces due to \( f' \neq 0 \);

– Often resembles the ruled quadrics.
Gröninger’s Case

- A special instantaneous motion:
  - Camera points downward, no roll
    \[ t \perp \hat{z}, \omega \perp t \]
  - Moving on a sphere while pointing to the center, or moving on a plane while pointing perpendicularly

- A special configuration of two such motions:
  - Known translation \( t_1 \parallel t_2 \)
  - Known yaw speed \( (\omega_1 - \omega_2) \cdot \hat{z} = 0 \)
  - Different pitch speed – the unknown
Simpler Surfaces

• Depth becomes a function of the radius

\[
Z_1 = \frac{2 (t_1 \cdot t_2) f' \neq 0}{\left( -(t_2 \cdot (\omega_2 \times \hat{z})) f^2 + (t_2 \cdot (\omega_1 \times \hat{z})) f \\
+ 2 (t_2 \cdot (\omega_1 \times \hat{z})) (p^T p) f' \right) / (t_1 \cdot t_2) Z_1}
\]

\[
Z_2 = \frac{(t_1 \cdot t_1) f - t_1 \cdot (\hat{z} \times (\omega_2 - f \omega_1)) Z_1}{(t_1 \cdot t_1) f - t_1 \cdot (\hat{z} \times (\omega_2 - f \omega_1)) Z_1}
\]

– Both are rotational symmetric surfaces
– Different surface curvatures (even signs)
– Does not exist without radial distortion!

Motion field $p'$

Two profile curves
Impact on Multi-view Reconstruction

- Persistent local ambiguity leads to accumulated error

- Synthetic captures with radial distortion
  - Capture#1 - plane
  - Capture#2 - Sphere

- Self-calibration using VisualSFM
  - Result#1
  - Result#2
In Real Life

To Thomas Gröninger:
– For your particular capture, the distortion cannot be solved by standard self-calibration

– Using camera calibration should resolve the problem

– (months later..) or, you try can change the motion pattern:
  • not always looking straight-down, or
  • not at constant height

From Thomas Gröninger:

Using approximate calibration

New capture & self-calibration
Recent Experimental Study


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<th>Straight-down</th>
<th>Forward-looking 5°</th>
<th>Straight-down + 20° banked views</th>
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<td><img src="image3.png" alt="Diagram" /></td>
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Conclusions

• Summary
  – Critical configurations for radial distortion self-calibration.
  – Radial distortion can be easily ambiguous (e.g. nadir capture).
    • Calibrate the camera, or alter the camera motion
    • Use additional motion priors in the reconstruction

• Future work
  – Extend the study to discrete viewpoints.

• Sincere thanks to Thomas Gröninger!
Questions?